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# On determination of mode II fracture toughness using semi-circular bend specimen

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## Abstract

The cracked semi-circular specimen subjected to three-point bending has been recognized as an appropriate test specimen for conducting mode I, mode II and mixed mode I/II fracture tests in brittle materials. The manufacturing and pre-cracking of the specimen are simple. No complicated loading fixture is also required for a fracture test. However, almost all of the theoretical criteria available for mixed mode brittle fracture fail to predict the experimentally determined mode II fracture toughness obtained from the semi-circular bend (SCB) specimen. In this paper, a modified maximum tangential stress criterion is used for calculating mode II fracture toughness  $K_{IIc}$  in terms of mode I fracture toughness  $K_{Ic}$ . The modified criterion is used for predicting the reported values of mode II fracture toughness for two brittle materials: a rock material (Johnstone) and a brittle polymer (PMMA). It is shown that the modified criterion provides very good predictions for experimental results.

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*Keywords:* Semi-circular bend specimen; Mode II fracture toughness; Stress intensity factor;  $T$ -stress; Brittle fracture

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## 1. Introduction

Crack extension in many brittle materials can take place when the crack is subjected to pure mode II. For such loading conditions, it is suggested that the crack propagates if the mode II stress intensity factor  $K_{II}$ , reaches the critical mode II stress intensity factor (or mode II fracture toughness,  $K_{IIc}$ ). Therefore, it is important to develop appropriate theoretical or experimental methods for determining mode II fracture toughness  $K_{IIc}$  in brittle materials. Unlike the mode I fracture toughness testing (ASTM E399-83; BS

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5447), no standard testing methods have been developed for determining the mode II fracture toughness. A number of specimens have been suggested in the past for determining the mode II fracture toughness of brittle materials.

The disc type test specimens such as the centrally cracked Brazilian disc (CCBD) specimen subjected to a diametral compressive load and the semi-circular specimen under three-point bending (SCB specimen) have been among favorite specimens for fracture tests in brittle materials (e.g. Awaji and Sato, 1978; Shetty et al., 1987; Atkinson et al., 1982; Chong and Kuruppu, 1984, 1987; Lim et al., 1994a,b; Krishnan et al., 1998; Khan and Al-Shayea, 2000; Chang et al., 2002). Mode II is provided for each of these two specimens by setting the crack line in an appropriate angle relative to the loading direction. The test procedures in both CCBD and SCB specimens are easy and cost effective and allow the determination of  $K_{Ic}$ ,  $K_{IIc}$  and a wide range of intermediate values between  $K_{Ic}$  and  $K_{IIc}$  for investigating the material behavior under complex states of loading.

In addition to the experimental techniques mentioned above, there are several theoretical criteria for predicting mixed mode I/II or pure mode II brittle fracture in cracked bodies. The maximum tangential stress (MTS) criterion (Erdogan and Sih, 1963), the minimum strain energy density criterion (Sih, 1973) and the maximum energy release rate criterion (Hussain et al., 1974) are three most important criteria for brittle fracture in mixed mode loading. Among these criteria, the maximum tangential stress criterion has been more popular because of its simplicity. Theoretical predictions based on this criterion suggests that the ratio of mode II fracture toughness  $K_{IIc}$  over mode I fracture toughness  $K_{Ic}$  is a fixed value as

$$\frac{K_{IIc}}{K_{Ic}} = 0.87 \quad (1.1)$$

However, there are many experimental investigations reporting a ratio of  $K_{IIc}/K_{Ic}$  significantly higher or lower than 0.87. A review of the related papers shows that depending on the tested material and the specimen geometry, the ratio of  $K_{IIc}/K_{Ic}$  varies typically from 0.45 to 2.2. For example, those available experimental results obtained from fracture tests on the semi-circular bend (SCB) specimen display a ratio of  $K_{IIc}/K_{Ic}$  considerably less than 0.87 (e.g. Lim et al., 1994b). Such inconsistencies between the theoretical and experimental results restrict the validity of fracture criteria like the maximum tangential stress criterion to limited geometry and loading conditions.

In this paper, a modified maximum tangential stress criterion is used for predicting brittle fracture in mode II loading. The modified MTS criterion takes into account the effect of non-singular stress term ( $T$ -stress) in addition to the singular stress term. Then, the theoretical predictions calculated from the modified MTS criterion are verified by using some available experimental results obtained from mode II tests on the semi-circular bend (SCB) specimen.

## 2. The semi-circular bend (SCB) specimen

In this section, first the semi-circular bend (SCB) specimen is described briefly and then the crack tip parameters of the specimen are presented. Chong and Kuruppu (1984) were among the first who suggested this specimen for conducting fracture tests on rock materials. Since then, the SCB specimen has been used frequently to investigate mixed mode fracture for brittle materials. The specimen has a simple geometry and can be prepared from typical rock cores. Little machining operations and easy test set-up procedure with common fracture toughness testing apparatus can be considered as major advantages of the SCB specimen.

As shown in Fig. 1 the specimen is a semi-circular disc of radius  $R$  with an angled edge crack of length  $a$  manufactured from the center of the semi-circle. Fracture tests are performed by loading the specimen under three-point bending. The mode I and mode II stress intensity factors  $K_I$  and  $K_{II}$  for the SCB specimen are often written as

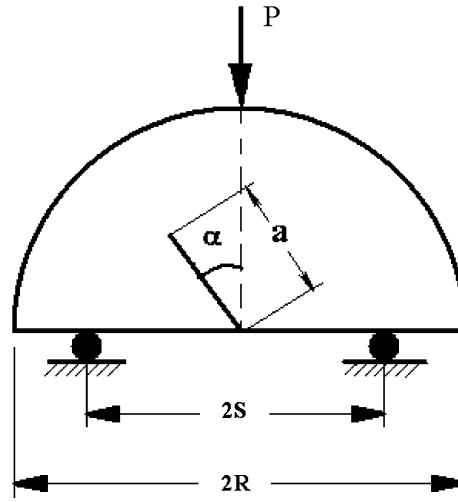
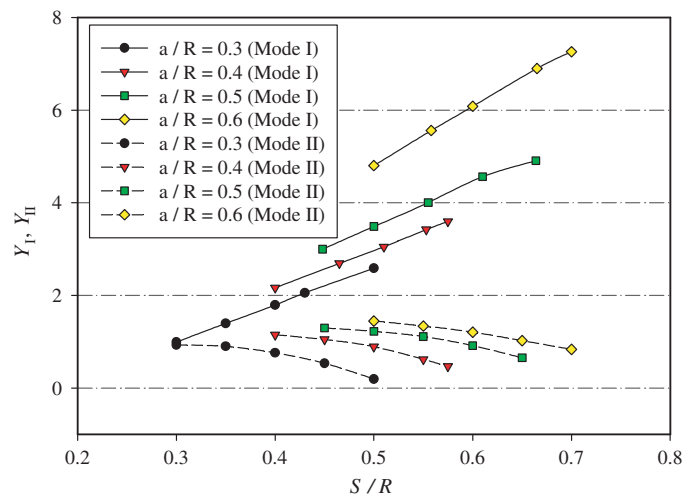


Fig. 1. Semi-circular bend (SCB) specimen.

$$K_I = \frac{P\sqrt{\pi a}}{2Rt} Y_I \left( \alpha, \frac{a}{R}, \frac{S}{R} \right) \quad (2.1)$$

$$K_{II} = \frac{P\sqrt{\pi a}}{2Rt} Y_{II} \left( \alpha, \frac{a}{R}, \frac{S}{R} \right) \quad (2.2)$$

where  $P$  is the compressive applied load and  $t$  is the thickness of specimen. The mode I and mode II geometry factors  $Y_I$  and  $Y_{II}$  are functions of crack length ratio  $a/R$ , half span to radius ratio  $S/R$  and the crack angle  $\alpha$  (that is the angle between the crack line and the vertical direction). Different combinations of mode I and mode II can be provided by changing the crack angle  $\alpha$  and the ratios  $a/R$  and  $S/R$ . Several researchers have used either analytical or numerical techniques for calculating the factors  $Y_I$  and  $Y_{II}$  (e.g. Kuruppu and Chong, 1986; Lim et al., 1993; Ayatollahi and Aliha, 2004). Fig. 2 shows the finite element results for

Fig. 2. Geometry factors  $Y_I$  and  $Y_{II}$  in pure mode I and pure mode II for different values of  $a/R$  and  $S/R$ .

the values of  $Y_I$  and  $Y_{II}$  corresponding to pure mode I and pure mode II, respectively, for various values of  $a/R$  and  $S/R$  (Ayatollahi and Aliha, 2004). When  $\alpha$  is zero, the specimen is always subjected to pure mode I. However, mode II is achieved at different angles  $\alpha$ , depending on  $a/R$  and  $S/R$ . Fig. 3 displays the values of  $\alpha$  corresponding to pure mode II in the SCB specimen for various ratios of  $a/R$  and  $S/R$ . For higher values of  $\alpha$  the mode I stress intensity factor becomes negative. It is noted that Ayatollahi and Aliha (2004) employed the properties of the path independent  $J$ -integral for calculating the stress intensity factors. The  $J$ -integral was obtained directly from ABAQUS, which uses the modified virtual crack extension method proposed by Li et al. (1985).

Recent studies by Ayatollahi and his co-workers (Smith et al., 2001; Ayatollahi et al., 2002; Ayatollahi and Aliha, 2004) have revealed that in addition to the stress intensity factors  $K_I$  and  $K_{II}$ , the non-singular

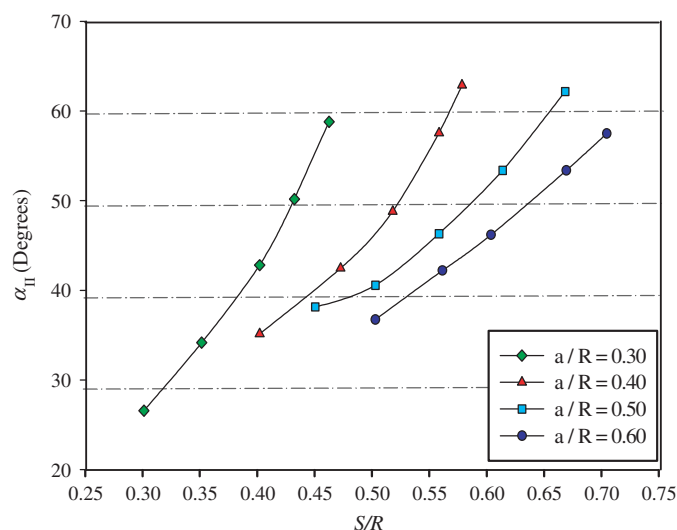


Fig. 3. The crack angle  $\alpha$  corresponding to pure mode II for different values of  $a/R$  and  $S/R$ .

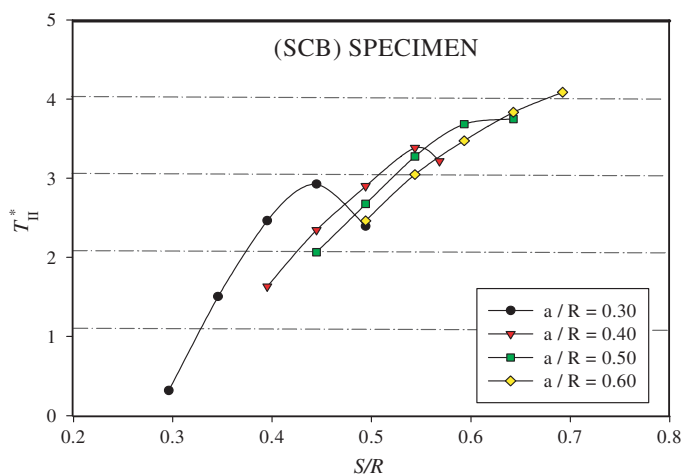


Fig. 4. The normalized parameter  $T_{II}^*$  in pure mode II for different values of  $a/R$  and  $S/R$ .

stress term (often known as  $T$ -stress) has significant effects on brittle fracture in pure mode II and mixed mode I/II loading. The  $T$ -stress has been calculated for the semi-circular bend specimen by Ayatollahi and Aliha (2004) using the finite element method and a stress difference technique (Ayatollahi et al., 1998). They made use of a dimensionless parameter

$$T^* \left( \alpha, \frac{a}{R}, \frac{S}{R} \right) = T \frac{2Rt}{P} \quad (2.3)$$

and derived the numerical values of  $T^*$  for different combinations of mode I and mode II. Fig. 4, shows the variations of  $T^*$  versus the ratio  $S/R$  and for various crack length ratios  $a/R$  for pure mode II.

### 3. Mode II brittle fracture

The numerical values of  $K_I$ ,  $K_{II}$  and  $T$  presented in the previous section are used in this section for predicting mode II brittle fracture in the SCB specimen. The theoretical calculations are based on a modified maximum tangential stress criterion which is described below. The elastic tangential stress in mixed mode loading can be written as

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos^2 \frac{\theta}{2} \left[ K_I \cos \frac{\theta}{2} - 3K_{II} \sin \frac{\theta}{2} \right] + T \sin^2 \theta + O(r^{1/2}) \quad (3.1)$$

where  $r$  and  $\theta$  are the conventional crack tip co-ordinates. As mentioned earlier,  $T$  is a constant term independent of distance  $r$  from the crack tip. The higher order terms  $O(r^{1/2})$  are usually negligible near the crack tip.

The maximum tangential stress (MTS) criterion was first proposed by Erdogan and Sih (1963) for brittle fracture in mixed mode I/II crack problems. According to this criterion, crack growth initiates radially from the crack tip along the direction of maximum tangential stress  $\theta_m$ . Also the crack extension takes place when the tangential stress  $\sigma_{\theta\theta}$  along  $\theta_m$  and at a critical distance  $r_c$  from the crack tip attains a critical value  $\sigma_{\theta\theta_c}$ . Both  $r_c$  and  $\sigma_{\theta\theta_c}$  are assumed to be material properties. The criterion in general can be used for pure mode I, pure mode II and any combinations of mode I and mode II loading. The conventional MTS criterion as proposed by Erdogan and Sih (1963) takes into account only the singular term in Eq. (3.1). Here in this research, the MTS criterion is used only for investigating pure mode II fracture. It is also modified such that both the singular term and the  $T$ -stress are considered in the tangential stress. For pure mode II,  $K_I$  is zero and Eq. (3.1) is written as

$$\sigma_{\theta\theta} = \frac{-3K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos^2 \frac{\theta}{2} + T \sin^2 \theta + O(r^{1/2}) \quad (3.2)$$

By ignoring the effects of higher order terms  $O(r^{1/2})$  the angle of maximum tangential stress  $\theta_m$  is determined from

$$\left. \frac{\partial \sigma_{\theta\theta}}{\partial \theta} \right|_{\theta=\theta_m} = \cos \theta_m \left( \frac{16T\sqrt{2\pi r_c}}{K_{II}} \sin \frac{\theta_m}{2} - 9 \right) + 3 = 0 \quad (3.3)$$

According to Eq. (3.3), the angle of maximum tangential stress in mode II depends on the  $T$ -stress. Once the angle  $\theta_m$  is calculated for given values of  $K_{II}$ ,  $T$  and  $r_c$  from Eq. (3.3), the onset of mode II brittle fracture can be predicted based on the second hypothesis of the maximum tangential stress criterion. This hypothesis suggests that brittle fracture occurs when

$$\sigma_{\theta\theta}(r_c, \theta_m) = \sigma_{\theta\theta_c} \quad (3.4)$$

For pure mode II, Eq. (3.4) gives

$$-3K_{IIc} \sin \frac{\theta_m}{2} \cos^2 \frac{\theta_m}{2} + \sqrt{2\pi r_c} T_c \sin^2 \theta_m = \sqrt{2\pi r_c} \sigma_{\theta\theta_c} \quad (3.5)$$

where  $K_{IIc}$  and  $T_c$  are the critical values of mode II stress intensity factor and  $T$  corresponding to the fracture load.  $K_{IIc}$  is also called mode II fracture toughness.

For pure mode I brittle fracture,  $K_{II} = 0$ ,  $\theta_m = 0$  and  $K_I = K_{Ic}$ . Therefore, Eqs. (3.1) and (3.4) yield to

$$\sigma_{\theta\theta_c} = \frac{K_{Ic}}{\sqrt{2\pi r_c}} \quad (3.6)$$

If the angle  $\theta_m$  determined from Eq. (3.3), and the critical stress  $\sigma_{\theta\theta_c}$  defined by Eq. (3.6), are replaced in Eq. (3.5) the fracture toughness ratio  $K_{IIc}/K_{Ic}$  can be determined from

$$\frac{K_{IIc}}{K_{Ic}} = \frac{1}{\sin \theta_m \left( \sqrt{2\pi r_c} \frac{T_c}{K_{IIc}} \sin \theta_m - \frac{3}{2} \cos \frac{\theta_m}{2} \right)} \quad (3.7)$$

Eq. (3.7) shows that based on the modified MTS criterion the fracture toughness ratio is a function of  $\frac{T_c}{K_{IIc}}$  and  $r_c$ . However, because both  $T_c$  and  $K_{IIc}$  are directly proportional to the fracture load,  $\frac{T_c}{K_{IIc}}$  can be simplified as

$$\frac{T_c}{K_{IIc}} = \frac{T^*}{Y_{II} \sqrt{\pi a}} = \frac{T^{**}}{\sqrt{\pi a}} \quad (3.8)$$

The dimensionless parameter  $T^{**}$  is shown in Fig. 5 for different values of  $a/R$ . Eq. (3.8) indicates that  $\frac{T_c}{K_{IIc}}$  is independent of the load and is only a function of the crack length  $a$  and the specimen radius  $R$ . Eqs. (3.3) and (3.7) can be now rewritten in terms of  $T^{**} \sqrt{\frac{2r_c}{a}}$ , respectively, as

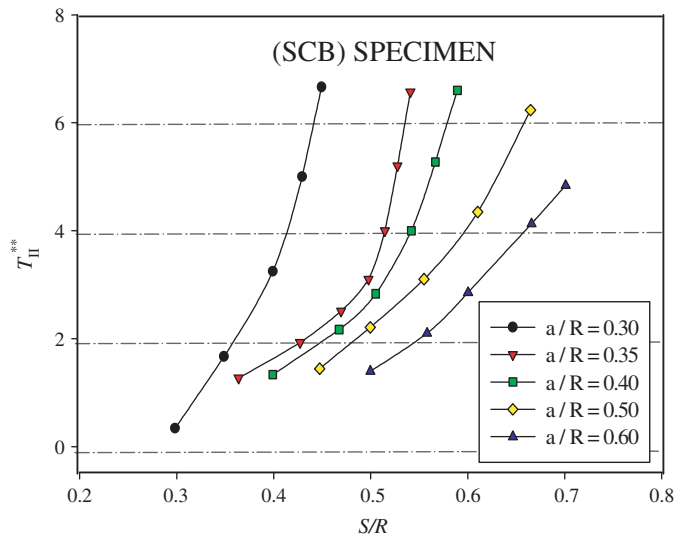


Fig. 5. The dimensionless parameter  $T^{**}$  in pure mode II for different values of  $a/R$  and  $S/R$ .

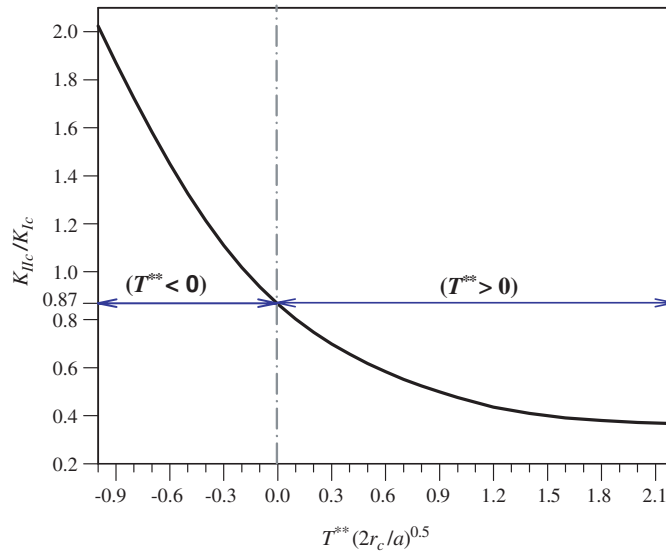


Fig. 6. The fracture toughness ratio  $K_{IIc}/K_{Ic}$  in the SCB specimen predicted using the modified MTS criterion.

$$16T^{**} \cos \theta_m \left( \sqrt{\frac{2r_c}{a}} \sin \frac{\theta_m}{2} - 9 \right) + 3 = 0 \quad (3.9)$$

and

$$\frac{K_{IIc}}{K_{Ic}} = \frac{1}{\sin \theta_m \left( T^{**} \sqrt{\frac{2r_c}{a}} \sin \theta_m - \frac{3}{2} \cos \frac{\theta_m}{2} \right)} \quad (3.10)$$

For any given value of  $T^{**} \sqrt{\frac{2r_c}{a}}$ , the angle  $\theta_m$  can be calculated using Eq. (3.9). By replacing these two values in Eq. (3.10), the fracture toughness ratio  $K_{IIc}/K_{Ic}$  is determined in terms of  $T^{**} \sqrt{\frac{2r_c}{a}}$ . The related numerical results have been plotted in Fig. 6. It is seen from Fig. 6 that the mode II fracture toughness is lower for specimens having a larger  $T^{**}$ . This implies that the mode II fracture toughness is not merely a material property but depends on the geometry of specimen and the type of loading as well.

#### 4. Discussion

The conventional MTS criterion (Erdogan and Sih, 1963) suggests that the angle of fracture initiation in pure mode II is  $-70.5^\circ$  and the fracture toughness ratio  $K_{IIc}/K_{Ic}$  is always equal to 0.87. However, according to the modified MTS criterion the fracture toughness ratio  $K_{IIc}/K_{Ic}$  is 0.87 only for mode II specimens in which the  $T$ -stress is zero. For specimens having a positive  $T$ -stress in mode II, the value of  $K_{IIc}/K_{Ic}$  is lower than 0.87 and for specimens having a negative  $T$ -stress, the value of  $K_{IIc}/K_{Ic}$  is higher than 0.87 (see Fig. 6).

The available experimental results obtained from fracture tests on the SCB specimen show that the fracture toughness ratio,  $K_{IIc}/K_{Ic}$  is significantly lower than 0.87. For example, Lim et al. (1994a,b) conducted a series of fracture tests on a water-saturated synthetic mudstone called *Johnstone* using the SCB test configuration. The tests were carried out under mode I, mode II and mixed mode I/II loading conditions. Lim

et al. (1994a,b) investigated the effects of different parameters such as the diameter and thickness of specimen, the crack length, the ratio of  $S/R$  and the saturated water content on fracture toughness of tested material. The pure mode II was obtained in their tests at the angle of crack inclination  $\alpha = 54^\circ$  and by considering  $S/R = 0.5$ ,  $a \approx 16.5$  mm,  $R \approx 48$  mm and  $w \approx 17.7\%$  (where  $w$  is the percent of saturated water content). The average value of mode II fracture toughness  $K_{IIc}$  obtained from the results of 3 tests was approximately  $0.935 \text{ MPa}\sqrt{\text{mm}}$ . Lim et al. (1994a) also performed a large number of mode I tests at  $\alpha = 0^\circ$  using various values of specimen diameter  $2R$ ,  $a/R$  and  $S/R$ . For calculating the fracture toughness ratio  $K_{IIc}/K_{Ic}$ , it is important to take  $K_{Ic}$  from the results of only those mode I tests which have conditions similar to already described mode II tests. Therefore,  $K_{Ic}$  is determined here by averaging the results of seven mode I tests having  $S/R = 0.5$ ,  $a \approx 16.5$  mm,  $R \approx 48$  mm and  $w \approx 17.7\%$  (Lim et al., 1994a). The average value for mode I fracture toughness is found to be  $K_{Ic} = 2.2 \text{ MPa}\sqrt{\text{mm}}$ . The fracture toughness ratio  $K_{IIc}/K_{Ic}$  for the SCB specimens made of Johnstone is thus about 0.42, which is considerably less than the figure 0.87 suggested by the conventional MTS criterion.

The fracture toughness ratio  $K_{IIc}/K_{Ic}$  for a brittle polymer PMMA (Perspex) has also been obtained using the SCB specimen (Aliha, 2004). The fracture tests were conducted on SCB specimens of  $a/R = 0.30$  and  $S/R = 0.43$ , and pure mode II could be provided at the crack angle  $\alpha = 50^\circ$ . Aliha (2004) has reported that the average value of  $K_{IIc}/K_{Ic}$  for PMMA is about 0.542. The experimental results presented in Lim et al. (1994b) and Aliha (2004) show that the values of  $K_{IIc}/K_{Ic}$  obtained from fracture test on the SCB specimen are significantly lower than 0.87 for both Johnstone and PMMA.

Fig. 5 shows that in a mode II SCB specimen, the  $T$ -stress is always positive for any combinations of  $a/R$  and  $S/R$ . Meanwhile, the modified MTS criterion proposes that  $K_{IIc}/K_{Ic}$  is lower than 0.87 if the  $T$ -stress in mode II is positive (see Fig. 6). Therefore, the reduction in the fracture toughness ratio  $K_{IIc}/K_{Ic}$  for the SCB specimen can be justified by the modified MTS criterion. Fig. 6, indicates that for calculating the fracture toughness ratio  $K_{IIc}/K_{Ic}$ , three parameters are required  $T^{**}$ ,  $r_c$  and  $a$ . While  $a$  is available from the specimen geometry, the parameter  $T^{**}$  is obtained from numerical results presented in Fig. 5. For some brittle materials, the critical distance  $r_c$  is often approximated by the radius of fracture process zone. Schmidt (1980) proposed a maximum normal stress criterion to predict the size of fracture process zone in rocks as

$$r_c = \frac{1}{2\pi} \left( \frac{K_{Ic}}{\sigma_t} \right)^2 \quad (4.1)$$

where  $\sigma_t$  is the tensile strength. By replacing  $K_{Ic} = 2.2 \text{ MPa}\sqrt{\text{mm}}$  and  $\sigma_t = 0.435 \text{ MPa}$  (for  $w \approx 17.7\%$ ) (Lim et al., 1994a) in Eq. (4.1), the value of  $r_c$  is estimated for Johnstone to be 4 mm. Based on the results of earlier fracture tests reported by Ayatollahi et al. (2002) the critical radius  $r_c$  for PMMA is about 0.1 mm.

Table 1 displays the details of procedure used for theoretical calculations of the fracture toughness ratio  $K_{IIc}/K_{Ic}$  for Johnstone and PMMA. For Johnstone, the theoretical predictions based on the modified MTS criterion suggests that  $K_{IIc}/K_{Ic} = 0.38$  that is consistent well with the average of the experimental results  $K_{IIc}/K_{Ic} = 0.42$  as described above. For PMMA, the predicted value of  $K_{IIc}/K_{Ic}$  is 0.588 and the average of experimental results is 0.542. Again it is seen that a good agreement exists between the experimental

Table 1

Fracture toughness ratio  $K_{IIc}/K_{Ic}$  predicted from the modified MTS criterion

Material	$r_c$ (mm)	$a$ (mm)	$R$ (mm)	$S/R$	$T^{**}$ (from Fig. 5)	$T^{**} \sqrt{\frac{2r_c}{a}}$	$K_{IIc}/K_{Ic}$ (predicted by MTS criterion)	$K_{IIc}/K_{Ic}$ (predicted by Modified MTS criterion)	$K_{IIc}/K_{Ic}$ (average of test results)
Johnstone (Lim et al., 1994a,b)	4.0	16.5	47.4	0.50	+3.04	2.100	0.87	0.371	0.42
PMMA (Aliha, 2004)	0.1	15	50	0.43	+5.02	0.585	0.87	0.588	0.542



results and the theoretical predictions when the modified MTS criterion is used. The average value for the angle of initiation of fracture for PMMA has been reported by Aliha (2004) to be about  $-82.5^\circ$ . Using  $r_c = 0.1$  mm, this angle is predicted by the modified MTS criterion (Eq. (3.3)) to be  $-78^\circ$  which in comparison with the angle  $-70.5^\circ$  suggested by the conventional MTS criterion, is much closer to the average of the experimentally observed fracture initiation angles. It is noteworthy that the numerical values of fracture initiation angle have not been given for Johnstone by Lim et al. (1994b).

The available criteria for mixed mode brittle fracture usually fail to predict the values of mode II fracture toughness  $K_{IIc}$  which are obtained experimentally from the SCB test specimen or some other test specimens. Therefore, some of the researchers have suggested empirical criteria to fit the experimental results (e.g. Lim et al., 1994b; Chang et al., 2002). In these criteria, a curve (often in the form of an elliptical curve) is fitted to the mixed mode test data. The experimental results are obtained from a series of fracture tests conducted in the whole range between pure mode I and pure mode II. Then, mode II fracture toughness of the tested material is determined from the empirical fitted curve. However, mode II fracture toughness and the ratio  $K_{IIc}/K_{Ic}$  in the empirical curves are significantly dependent on the geometry and loading configurations. For example, the fracture toughness ratio  $K_{IIc}/K_{Ic}$  obtained from fracture tests on the Brazilian disc specimen is much higher than that obtained from the SCB specimen. This implies that there is no unique empirical curve for a given material. Furthermore, the prediction of mode II brittle fracture using the empirical curves requires an inevitable set of mode II fracture tests, whereas the modified MTS criterion can provide a reasonable prediction for mode II fracture toughness from extensive data available for mode I fracture toughness of different materials.

A series of computational studies by Ayatollahi and his co-researchers (Ayatollahi et al., 1996, 1998; Ayatollahi and Aliha, 2005) show that similar to the SCB specimen, there are several other mode II specimens in which significant values of  $T$ -stress exist in conjunction with the  $K_{II}$  term of stress. The modified MTS criterion reveals that for such specimens, the  $T$ -stress influences considerably the magnitude of mode II fracture toughness in predominantly linear elastic materials. Therefore, according to the results obtained in this paper and those presented earlier (Ayatollahi et al., 1996, 1998, 2002; Smith et al., 2001), ignoring the effects of  $T$ -stress in mode II specimens may yield a great discrepancy between the experimental results and the theoretical predictions. It should be noted that the  $T$ -stress and  $r_c$  have already been used by researchers (e.g. Williams and Ewing, 1972; Pettit et al., 2001) for a better prediction of experimental results in mixed mode fracture. However, previous studies are mainly confined to angled central crack problem subjected to uniaxial loading. Since this specimen is not able to provide pure mode II, the effect of  $T$ -stress on mode II fracture toughness has not been received enough attention in the past.

The theoretical findings of this research suggest that the SCB specimen always underestimates the value of mode II fracture toughness in engineering materials. This is because a large positive  $T$ -stress is present in the SCB specimen when the specimen is subjected to mode II. However, the  $T$ -stress in real applications is rarely as positive as in the SCB specimen. Since the fracture resistance of a given material in real applications (i.e. other than the test conditions) can be significantly higher than that determined from fracture tests on the SCB specimen, this specimen provides a conservative prediction for mode II fracture toughness in brittle materials.

## 5. Conclusions

- (1) The non-singular stress term ( $T$ -stress) in the semi-circular bend (SCB) specimen is always positive for any combinations of  $a/R$  and  $S/R$  when the specimen is subjected to pure mode II.
- (2) A modified MTS criterion was used to predict the fracture toughness ratio  $K_{IIc}/K_{Ic}$ . According to this criterion, the ratio  $K_{IIc}/K_{Ic}$  in the SCB specimen is considerably lower than 0.87 proposed by the conventional MTS criterion. This prediction is in good agreement with those available experimental results obtained from fracture tests using the SCB specimen.

- (3) It is expected the SCB specimen always provide a conservative prediction for mode II fracture toughness in brittle materials.

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